

Excess magneto-resistance in multiband superconductors due to the viscous flow of composite vortices

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By using the time-dependent Ginzburg-Landau theory, we show that extremely diverse experimental data on flux-flow resistivity in multiband superconductors can be qualitatively explained by a composite nature of Abrikosov vortices consisting of elementary fractional vortices in different bands. In composite vortices, the ratio of a core size to electric field relaxation length is found to vary in wide limits depending on system parameters. As a result, the flux-flow magneto-resistance can strongly exceed the single-component Bardeen-Stephen estimation provided that moving vortices generate electric field stretching strongly outside the vortex cores.

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Recent experimental studies of resistive states in multiband superconductors revealed unusual vortex physics associated with the viscous flow of magnetic flux. The magnetic field dependencies of flux-flow resistivity ρ_f were found to be qualitatively different from the single-band behaviour established in classical experiments [1] and theoretical works by Bardeen and Stephen [2], Tinkham [3], and Nozieres and Vinen [4].

In conventional type-2 superconductors at low temperatures $T \ll T_c$ the flux-flow resistivity is well described by a linear Bardeen-Stephen (BS) law $\rho_f/\rho_n = \gamma \mathcal{B}/H_{c2}$, where ρ_n is the normal state resistivity, \mathcal{B} is an average magnetic induction, H_{c2} is the second critical field and $\gamma \approx 1$ is the magneto-resistance coefficient [1]. At elevated temperatures $T \rightarrow T_c$, the vortex motion is strongly impeded due to the enhanced electron-phonon relaxation which results in a significant suppression of ρ_f/ρ_n so that magneto-resistance defined by asymptotic at low magnetic fields decreases below the BS value. In the gapless regime $\gamma \approx 0.69$ [5]. Similarly, the vortex motion becomes more viscous due to the depairing effects resulting from spin-flip scattering at magnetic impurities. In this case $\gamma = 0.33$ [6].

In contrast to the conventional single-band behaviour, multiband superconductors MgB_2 [7] and $Li111$ [8, 9] were found to have the flux-flow magneto-resistance increased above BS-value line $\gamma > 1$ so that $\rho_f/\rho_n > \mathcal{B}/H_{c2}$. The experimentally found dependencies $\rho_f(\mathcal{B})$ have a steeper growth in the low-field region with an enhanced magneto-resistance $\gamma \approx 1.4$. For $P-Ba122$ [10] and $P-Sr122$ [11] systems, even larger slopes were obtained $\gamma \approx 2.5$. Moreover, the opposite behaviour of smaller magneto-resistance was also observed in experiments with $FeSeTe$, that reported $\gamma \approx 0.7$ [12].

In this paper we show that the enhanced flux-flow magneto-resistance can be explained by a composite nature of vortices in multiband superconductors. These complex topological excitations consist of several singularities corresponding to phase windings of components of the or-

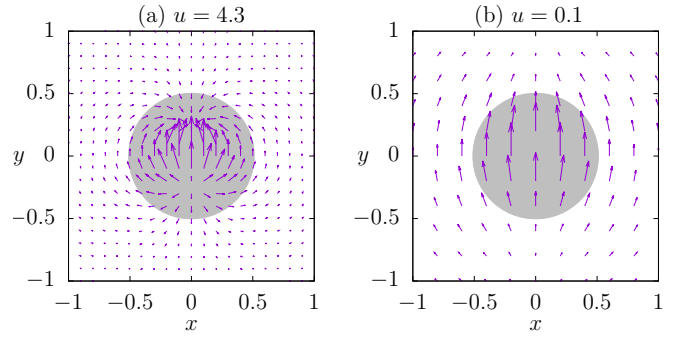


FIG. 1: Distribution of electric field (vectors) in a two-band superconductor around composite vortex line oriented along the z axis. Vortex core size (gray area) is defined by the boundary where the gaps recover 0.95% of the bulk value. The panels differ only in the ratio of diffusion coefficients (a) $D_2/D_1 = 0.5$, $u = 4.3$ and (b) $D_2/D_1 = 70$, $u = 0.1$. The parameter u is defined in text.

der parameter in different superconducting bands. Such composite objects can be considered as bound states of several fractional vortices [13]. In most cases the equilibrium state corresponds to co-centred fractional vortices although they can split under the action of fluctuations [14, 15], external drive [16] or due to the interaction with other vortices and sample boundaries [17, 18].

The viscous motion of composite vortices under the action of an external Lorentz force is determined simultaneously by the non-equilibrium processes in several superconducting bands. The presence of additional conducting bands with smaller gap amplitudes increases the total density of normal electrons trapped within vortex cores. This weakens the screening of electric field, which can stretch out of the vortex core at distances far exceeding the coherence length, cf panels in Fig. 1. As a result of such a non-trivial interplay of the vortex core size and the electric field relaxation length, the value of magneto-resistance γ can change in wide interval. Such a behaviour is in high contrast to single-band supercon-

ductors characterized in the gapless regime by a universal value $\gamma \approx 0.69$ [5].

The non-equilibrium processes related to the vortex motion are known to be rather diverse. Different dissipation mechanisms can play the dominating role depending on the range of temperatures and magnetic fields. In this paper we consider an *s*-wave superconductor [36] at temperatures in the immediate vicinity of the critical one given by $(T_c - T) \ll \hbar\tau_{\text{ph}}^{-1}$, where τ_{ph}^{-1} is the electron-phonon scattering rate. Within such temperature interval, gapless superconducting states are realized as a result of the inelastic interactions with phonons.

Non-equilibrium effects in gapless superconductors can be described by the time-dependent Ginzburg-Landau (TDGL) theory. We start from the microscopic weak-coupling model of a dirty two-band superconductor described by the 2×2 matrix of coupling constants $\hat{\Lambda} = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix}$ and diffusion coefficients D_k in each superconducting band ($k = 1, 2$). We derive TDGL generalization for a two-band superconductor formulated in terms of the two-component complex field (ψ_1, ψ_2) , see Supplementary Material.

The two-band Ginzburg-Landau model is an expansion in several small parameters [19]: small gaps and gradients, not to be confused with the expansion in single small parameter $\tau = 1 - T/T_c$. For conditions when it holds and accurately approximates microscopic theory see [20]. The subdominant component can alter substantially the magnetic properties [20–24]. However, for ordinary *s*-wave superconductors that undergo a single second-order phase transition, the parameter τ must become the smallest one sufficiently close to T_c since the system breaks only $U(1)$ symmetry. Then the model is well approximated by a single-component Ginzburg-Landau theory [25]. In this paper we are interested in the regime very close to T_c . Then the second component can be excluded by projecting the fields $\psi_{1,2}$ to the eigenvector corresponding to the maximal eigenvalue of the coupling matrix $\hat{\Lambda}$, see detailed discussion in [20]. To implement such a projection we use the ansatz $(\psi_1, \psi_2)^T = \psi(\mu, 1)^T$, where $\mu = (\lambda_- + \lambda_0)/(2\lambda_{21})$, $\lambda_- = \lambda_{11} - \lambda_{22}$ and $\lambda_0 = \sqrt{\lambda_- + 4\lambda_{12}\lambda_{21}}$. Here the common complex field ψ is a Landau order parameter corresponding to a $U(1)$ symmetry breaking in the two-band system.

Applying the transformation outlined above, we obtain the effective single-component TDGL equation for the two-band system

$$\Gamma(\partial_t + 2ie\varphi/\hbar)\psi = -\delta\psi^*\mathcal{F}, \quad (1)$$

$$\mathcal{F} = \int d^3\mathbf{R} \left(\alpha|\psi|^2 + K|(\nabla - 2\pi i\mathbf{A}/\phi_0)\psi|^2 + \frac{\beta}{2}|\psi|^4 \right).$$

Here φ , \mathbf{A} are scalar and vector potentials, ϕ_0 is flux

quantum and expansion coefficients are given by

$$\alpha = -\nu\tau, \quad \beta = 7\zeta(3)(\nu_2 + \nu_1\mu^4)/(8\pi^2T_c^2),$$

$$K = \pi\hbar\nu D/(8T_c), \quad \Gamma = \pi\hbar\nu/(8T_c), \quad (2)$$

where ν_k is density of states in each band, $\nu = \nu_2 + \nu_1\mu^2$ and $D = (\nu_2D_2 + \nu_1D_1\mu^2)/\nu$. In this approximation the order parameter is a composite field that describes joint contribution of two superconducting bands.

The non-equilibrium response described by Eq. (1) is determined by the parameter $u = \xi^2/l_e^2$ where ξ and l_e are the coherence length and electric field relaxation length, respectively. The value of coherence length $\xi = \sqrt{-K/\alpha}$ can be obtained from the equilibrium GL functional. The electric field relaxation length l_e can be found from the TDGL theory supplemented by the charge conservation law $\nabla \cdot (\mathbf{j}_s + \mathbf{j}_n) = 0$, where \mathbf{j}_s is the superconducting current and $\mathbf{j}_n = \sigma_n\mathbf{E}$ is the normal current given by electric field \mathbf{E} and normal state conductivity $\sigma_n = 2e^2 \sum_k \nu_k D_k$. By introducing gauge invariant scalar potential $\Phi = \varphi + \hbar\partial_t\theta/(2e)$, we write Eq. (1) in the form $\nabla \cdot \mathbf{j}_n = -8e^2\Gamma|\psi|^2\Phi/\hbar^2$ so that screening of scalar potential is described by equation $l_e^2\nabla^2\Phi = f^2\Phi$, where $l_e = \sqrt{\hbar^2\sigma_n/(8e^2\Gamma\psi_0^2)}$ and θ , f are order-parameter phase and its amplitude scaled by bulk value $\psi_0 = \sqrt{-\beta/\alpha}$.

In contrast to single-band superconductors which have the fixed value of $u \approx 5.79$ [26, 27], in multi-band compounds u strongly depends on the system parameters. Using the expressions for ξ and l_e obtained above together with the GL coefficients (2) we obtain

$$u = 5.79 \frac{(\nu_1\mu^2 + \nu_2)(\nu_1D_1\mu^2 + \nu_2D_2)}{(\nu_1\mu^4 + \nu_2)(\nu_1D_1 + \nu_2D_2)}. \quad (3)$$

As long as the diffusion coefficients $D_{1,2}$ can differ in orders of magnitude in realistic compounds [28–31], the latter expression can change in wide limits ranging from $u \ll 1$ to $u \gg 1$.

Let us consider several characteristic cases. First, we can assume that the interband pairing is weak $|\lambda_{12}| \ll \lambda_{kk}$. In this case $\mu \gg 1$ so that Eq. (3) yields $u = 5.79\nu_1D_1/(\nu_1D_1 + \nu_2D_2)$. This expression does not exceed the universal single-band numerical value realized when $\nu_2D_2 \rightarrow 0$. Qualitatively this result means that the electric field is screened less effectively due to presence of additional band with enhanced concentration of normal electrons. In the opposite case of interband-dominated pairing interaction provided that $\mu \sim 1$, the single-band value of u is recovered irrespective of the ratio of diffusion coefficients.

Resistive states of superconductors containing composite Abrikosov vortices is dramatically affected by an interplay between lengths ξ and l_e . The rate of energy dissipation induced by moving vortices is determined by the ratio of a vortex core size which is proportional to ξ

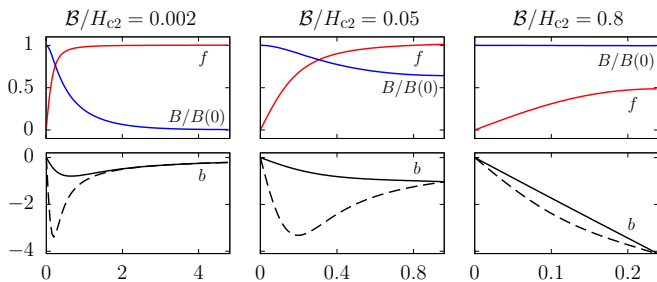


FIG. 2: (Color online) Upper row: The distributions $f(r)$ and $B(r)/B(0)$ for the order parameter and magnetic field is depicted for different vortex concentrations set by an average magnetic induction. Lower row: Distributions of scalar potential $b(r)$. For comparison, cases with $u = 0.1$ (black solid) and $u = 5.7$ (black dashed) are shown.

and the size of a domain where the electric field generated around moving vortex is concentrated. In the usual BS approach it is assumed that these scales are identical while the theory of gapless single-band superconductors with $u = 5.79$ determines stronger electric field localization. This leads to smaller values of flux-flow magneto-resistance $\gamma = 0.69$ as compared to the BS estimation $\gamma \approx 1$.

In multiband systems one can obtain qualitatively different regimes when the scale of electric field localization around moving composite vortices strongly exceeds the vortex core size. The resulting flux-flow resistivity can significantly exceed the BS estimate. We study this effect by calculating vortex structures and electric field created by moving vortices arranged in the regular lattice. To model the vortex array at finite values of magnetic induction we use standard circular cell approximation. In this approach the hexagonal unit cell of the triangular vortex lattice is replaced by a circle, where the order parameter and magnetic field distributions are assumed to be axially symmetric. The circular cell radius R_c is uniquely defined by averaged magnetic induction \mathcal{B} due to flux quantization $\pi R_c^2 = \phi_0/\mathcal{B}$. Several vortex structures calculated numerically by solving Eq. (1) supplemented with a Maxwell equation are shown in Fig. 2 for different values of magnetic induction. In these plots and below we normalize radial coordinate $R = r\lambda$ by the London penetration length $\lambda = \phi_0/\sqrt{-32\pi^3 K\alpha/\beta}$.

The stationary motion of vortices with a constant velocity \mathbf{U} is determined by the balance between Lorentz force on the vortex line $\mathbf{f}_{\text{ext}} = \mathbf{j}_{\text{tr}} \times \int \mathbf{B} d^2\mathbf{R}/c$ and a viscous friction $\mathbf{f}_{\text{env}} = -\eta\mathbf{U}$, where η is the vortex viscosity. To calculate η we use the TDGL theory assuming that the order parameter and magnetic field can be approximated by their equilibrium distributions transformed to the moving coordinate frame according to the Galilean transformation of the fields. To find the electric field generated by moving vortex array we employ again the circular cell approximation. By assum-

ing that $\mathbf{U} \parallel \mathbf{x}$, we take scalar potential in the form $\varphi(\mathbf{R}) = \phi_0 U \sin \theta b(r)/(2\pi c\lambda)$. Taking into account above mentioned equation describing screening of electric field, we arrive to the non-homogeneous linear equation for the scalar potential where the source term is generated by the moving vortex phase singularity

$$b'' + \frac{b'}{r} - \frac{b}{r^2} = u\kappa^2 f^2 \left(b + \frac{1}{r}\right), \quad (4)$$

where $\kappa = \lambda/\xi$ is a GL parameter. Equation is supplemented by the boundary conditions at $r = 0$ and $r = r_c$. The former results from the regularity criterion $b(0) = 0$. The latter can be obtained from the condition that the average electric field \mathcal{E} should satisfy a general relation $c\mathcal{E} = \mathcal{B} \times \mathbf{U}$. Using this constraint one gets boundary condition $b(r_c) = -1/r_c$. Eq. (4) defines a non-equilibrium electric response of a moving vortex. The problem is parametrized by only one parameter, u , which can change in wide interval controlling the electric field relaxation length relative to the vortex core size as illustrated in Fig. 2 where the distribution of b is shown for different magnetic fields and various values of u .

To discuss resistive state in multiband superconductor, we first analyze forces driving the motion of a single vortex. Following the general procedure outlined in [27], we obtain a general expression for the vortex viscosity within a circular cell approximation

$$\eta = 2\pi\Gamma\psi_0^2 \left[I_T + I_{BS} - \left(\frac{b'r + b}{u\kappa^2 r} \right)_{r_c} + \int_0^{r_c} \frac{a'^2 dr}{u\kappa^2 r} \right], \quad (5)$$

where $I_T = \int_0^{r_c} f^2 r dr$ and $I_{BS} = \int_0^{r_c} f^2 (b + 1/r) dr$. The last term in Eq. (5) contains the dimensionless magnetic flux $a(r) = \kappa^2 \int_0^r dr_1 r_1 B(r_1)/H_{c2}$. The flux-flow resistivity is given by the standard relation

$$\rho_f = \phi_0 \mathcal{B} / (c^2 \eta). \quad (6)$$

For dilute vortex lattices $\mathcal{B} \ll H_{c2}$ and $\kappa \gg 1$, the last two terms in Eq. (5) can be neglected. Therefore, for isolated vortices in extreme type-2 superconductors, the first two terms dominate. One of them I_T is the Tinkham's part characterizing the order parameter relaxation and the other I_{BS} takes into account Ohmic losses generated by normal currents flowing around moving vortices as discussed above. The former is fixed by an equilibrium vortex structure so that $I_T \approx 0.28$ while the value of I_{BS} is sensitive to parameter u .

In multiband superconductors where u can be made arbitrary small, I_{BS} can be strongly enhanced compared to the single-component case where $I_{BS} \approx 0.23$ [32]. As a result, the contribution of Ohmic losses to the overall vortex energy dissipation is enhanced which means that the environment becomes more viscous for moving composite vortices as compared to the single-component case.

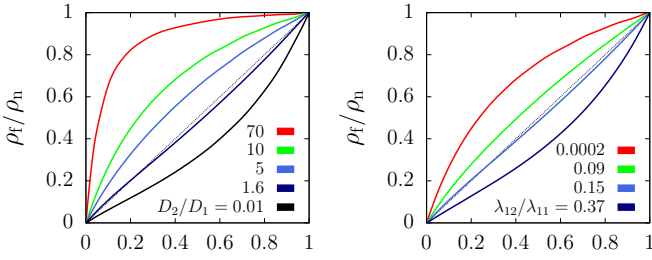


FIG. 3: (Color online) Flux-flow resistivity *vs* magnetic field. Left: As diffusion coefficient in the passive weaker band, D_2 , increases ($u = 5.7, 2.7, 1.3, 0.7, 0.1$, correspondingly). Right: As interband interaction constant λ_{12} increases ($u = 0.7, 1.7, 2.6, 4.9$, correspondingly). Thin dotted line is BS result.

For weak fields $\mathcal{B} \ll H_{c2}$ and large $\kappa \gg 1$, one gets

$$\frac{\rho_f}{\rho_n} = \frac{2}{u(I_T + I_{BS})} \frac{\mathcal{B}}{H_{c2}}. \quad (7)$$

The presence of parameter u in the denominator signals a possibility to enhance magneto-resistance γ due to the strong delocalization of electric field generated by composite vortices when $u \rightarrow 0$. This conclusion is confirmed by numerically calculated flux-flow resistivity curves. As shown in Fig. 3, the dependencies $\rho_f(\mathcal{B})$ can have much steeper slopes at $\mathcal{B} \ll H_{c2}$ as compared to the usual BS law shown by the dotted line.

Within the circular cell approximation, Eq. (7) can be applied to calculate the flux-flow resistivity at arbitrary fields $0 < \mathcal{B} < H_{c2}$. At that, denominator in Eq. (7) should be supplemented by the last two terms in Eq. (5). Such an approach is consistent with well-known analytical asymptote near H_{c2}

$$\frac{\rho_f}{\rho_n} = 1 - \frac{u\kappa^2}{\beta_A(2\kappa^2 - 1) + 1} \left(1 - \frac{\mathcal{B}}{H_{c2}}\right), \quad (8)$$

where $\beta_A = 1.16$ is Abrikosov parameter [33]. In single-band large- κ superconductors, the slope of the dependence (8) is $(H_{c2}/\rho_n)\partial_{\mathcal{B}}\rho_f = 2.5$ meaning that the curve $\rho_f(\mathcal{B})/\rho_n$ goes below the BS line. In the two-band case shown in Fig. 3, we obtain much more diverse behaviour in qualitative agreement with recent experimental data [8, 9] with enhanced magneto-resistance.

Energy dissipation by moving vortices is key limiting factor for practical applications of superconductivity. The diverse flux-flow behaviour of multiband superconductors has not been properly described by the existing theories developed for conventional single-band superconductors. Qualitatively the reason for disagreement was that the size of a non-equilibrium domain with localized electric field has been assumed to be proportional to the vortex core size, as in the pioneering work by Bardeen and Stephen. As we have demonstrated above this assumption is qualitatively incorrect for complex compos-

ite vortices in multiband superconductors formed by co-existing condensates in different superconducting bands. For these objects, an interplay of microscopic parameters such as diffusion coefficients and pairing constants in different bands can lead to the large variations in electric field relaxation length relative to the vortex core size. As a result, electric field can be stretched strongly outside the vortex cores dramatically enhancing the Ohmic losses and the overall energy dissipation by moving fluxes [37].

The unusual flux-flow phenomena that we discuss here is one of the possible examples of resistive states in superconductors. Broad range of non-stationary phenomena which should be strongly affected by multiband effects include the formation of resistive states in narrow superconducting channels such as the mesoscopic wires of the width compared to the coherence length [26, 34]. Such systems are quite important for technological applications in superconducting photon detectors [35]. Near critical temperature they have rather complicated behaviour strongly affected by the value of parameter u in TDGL theory. Different non-stationary regimes including the growth of critical superconducting nucleus and the formation of phase slip centres in homogeneous channels [34] and near the boundaries [26] have been thoroughly investigated for single-band compounds with the fixed value of $u = 5.79$. Generically different regimes with much smaller values of u become accessible in multiband superconductors reported in the present paper. This possibility opens potentially interesting directions of studying non-equilibrium current-carrying states in thin films and wires of multiband superconductors.

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- [37] Another mechanism which can affect flux-flow resistivity in multiband superconductors is splitting of moving composite vortices into a non-co-centered fractional ones under the action of an external current. This effect however should be diminished by interband Josephson coupling since fractional vortices attract each other linearly at the length scales larger than Josephson length (see e.g. [13]). In particular it should not be important in the region of our interest: near T_c because the Josephson length does not diverge when $T \rightarrow T_c$.